Roll No. .... Total Pages : 04

BT-I/D-22 **41046**

CALCULUS AND LINEAR ALGEBRA

BS-133A

Time : Three Hours] [Maximum Marks : 75

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

Unit I

1. (a) Prove that :

$$\beta\left(m, \frac{1}{2}\right) = 2^{2m-1}\beta(m, m)$$

(b) Verify Rolle's theorem for the function :

$$(x+2)^3(x-3)^4 \text{ in } (-2, 3)$$

2. (a) Evaluate :

$$\lim_{x\to 0} \frac{(1+x)^{1/x} - e - \frac{ex}{2}}{x^2}$$

(b) Find the volume of a sphere of radius *a*.

Unit II

3. (a) If $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}$ and I is the unit matrix of order 3, evaluate $A^2 - 3A + 9I$.

(b) Find the rank of the matrix :

$$\begin{bmatrix} 1 & 2 & 1 \\ -3 & -6 & -3 \\ 5 & 10 & 5 \end{bmatrix}$$

4. (a) Solve the following equations by Cramer's rule :

$$x+3y+6z=2$$
$$3x-y+4z=9$$
$$x-4y+2z=7$$

(b) Find the inverse of the matrix $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ and verify $A^{-1}A = I$, where I is the identity matrix of order 3.

Unit III

5. (a) Show that the set $v_1 = (2, -1, 0)$, $v_2 = (1, 2, 1)$ and $v_3 = (0, 2, -1)$ are linearly independent. Also express the vector (3, 2, 1) and (1, 1, 1) as a linear combination of v_1, v_2 and v_3.

(b) Prove that the set $\{(2, 1, 4), (1, -1, 2), (3, 1, -2)\}$ forms a basis of R^3.

6. (a) If $T : R^3 \to R^3$ be a linear operator defined by $T(x, y, z) = (2x, 4x - y, 2x + 3y - z)$, show that T is invertible.

(b) For what value of k (if any) the vector $v = (1, -2, k)$ can be expressed as a linear combination of vectors $v_1 = (3, 0, -2)$ and $v_2 = (2, -1, -5)$ in $R^3(R)$?

Unit IV

7. (a) Find the eigenvalues and eigenvectors of the matrix :

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

(b) Define orthogonal matrix and show that the following matrix is orthogonal :

$$\begin{bmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ \frac{1}{3} & -\frac{2}{3} & \frac{1}{3} \end{bmatrix}$$

8. (a) If α and β are vectors in an inner product space, then show that :

$$\|\alpha+\beta\|^2+\|\alpha-\beta\|^2=2\|\alpha\|^2+2\|\beta\|^2$$

(b) Define symmetric and skew-symmetric matrix and prove that every square matrix can be expressed as sum of a symmetric and skew-symmetric matrix.