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**BT–1/D-23** **41046**

# CALCULUS & LINEAR ALGEBRA

Paper–BS–133A

Time Allowed : 3 Hours] [Maximum Marks : 75

**Note** : Attempt **five** questions in all, selecting at least **one** question from each Unit. All questions carry equal marks.

## UNIT–I

1. (a) Express the integral $\int_0^1 \frac{dx}{\sqrt{1-x^4}}$ in terms of Gamma function.

   (b) Verify Rolle's Theorem for the function $(x-a)^m (x-b)^n$ where m, n are positive integers in $[a, b]$.

2. (a) Evaluate : $\lim_{x \to 0} \frac{(1+x)^{\frac{1}{x}} - e}{x}$.

   (b) Find the Volume formed by the Revolution of loop of the curve $y^2(a+x) = x^2(3a-x)$ about $x-axis$.

## UNIT–II

3\. (a) If $A=\begin{bmatrix}3 & 2\\2 & 3\end{bmatrix}$ and $I$ is the unit matrix of order 2, evaluate $A^2-6A+8I$.

(b) Find the rank of the matrix $\begin{bmatrix}3 & 4 & 1 & 2\\3 & 2 & 1 & 4\\7 & 6 & 2 & 5\end{bmatrix}$.

4\. (a) Solve the following equations by Cramer's rule.

$$x+y+z=4$$
$$x-y+z=0$$
$$2x+y+z=5.$$

(b) Find the inverse of the matrix $A=\begin{bmatrix}3 & 1 & 2\\2 & -3 & -1\\1 & 2 & 1\end{bmatrix}$ and verify $A^{-1}A=I$, where I is the identity matrix of order 3.

## UNIT–III

5\. (a) Show that the vectors (1,–2,1), (2,1,–1) and (7,–4,1) are linearly dependent in $R^3(R)$.

(b) Show that the set $\{(2,-1,0),(3,5,1),(1,1,2)\}$ forms a basis of $R^3$.

6. (a) State and Prove rank and nullity theorem.

(b) Let $T : R^3 \to R^3$ be a linear operator defined by $T(x, y, z) = (x+z, x-z, y)$, show that $T$ is invertible.

## UNIT–IV

7. (a) Find the eigen values and eigen vectors of the matrix

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}.$$

(b) If A is square matrix, show that :

(i) $A + A'$ symmetric.

(ii) $A - A'$ is skew-symmetric.

8. (a) Find the values $a$, $b$, $c$ if $A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$ is orthogonal.

(b) Let V(F) be an inner product space. If $u, v \in V$ such that $|\langle u, v \rangle| = \|u\| \cdot \|v\|$, then show that $u$ and $v$ are linear dependent.