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BT–1/D-24 **41046**

CALCULUS & LINEAR ALGEBRA

Paper–BS–133A

Time Allowed : 3 Hours] [Maximum Marks : 75

Note : Attempt **five** questions in all, selecting at least **one** question from each Unit. All questions carry equal marks.

UNIT–I

1. (a) Evaluate $\int_0^1 \frac{\sin^{-1} x}{x}\, dx$.

   (b) Evaluate $\int_0^\infty e^{-ax} x^{m-1} \sin bx\, dx$ in terms of Gamma function.

2. (a) Prove that $\beta(m, 1/2) = 2^{2m-1}\beta(m, m)$.

   (b) Evaluate $\lim_{x\to 0} \frac{(1+x)^{1/x} - 6}{x}$.

UNIT–II

3. (a) Find x, y, z and w given that :

$$3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 5 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 6 & x+y \\ z+w & 5 \end{bmatrix}.$$

(b) Determine the rank of the matrix :

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}.$$

4. (a) Using the Gauss-Jordan method, find the inverse of the matrix :

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}.$$

(b) Solve the equations :

$3x+y+2z=3,\ 2x-3y-z=-3,\ x+2y+z=4$ using Cramers's rule.

UNIT–III

5. (a) Determine the basis of row space for the matrix :

$$\begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{bmatrix}.$$ Also, find the dimension of the row space of the given matrix.

(b) Let $u_1=(1,1,-1), u_2=(4,1,1), u_3=(1,-1,2)$ be a basis of $\mathbb{R}^3$. Let $T:\mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation such that $T(u_1)=(1,0), T(u_2)=(0,1), T(u_3)=(1,1)$. Find T.

6. (a) Show that the map $T:\mathbb{R}^3 \to \mathbb{R}^2$ defined by $T(x,y,z) = (|x|, y-z)$ is not a linear transformation.

(b) Express the matrix $\begin{bmatrix} 2 & 0 \\ 4 & -5 \end{bmatrix}$ as a linear combination of the matrices $A = \begin{bmatrix} 0 & -3 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$.

UNIT–IV

7. (a) Find the Eigen values and Eigen vectors of the matrix

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}.$$

(b) If x and y are vectors in an inner product space V(F), then show that $x = y$ iff $\langle x,z \rangle = \langle y,z \rangle$, for all $z \in V$.

8. (a) Express the matrix A as the sum of a symmetric and a skew-symmetric matrix where :

$$A = \begin{bmatrix} 4 & 2 & -3 \\ 1 & 3 & -6 \\ -5 & 0 & -7 \end{bmatrix}.$$

(b) Show that the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is Orthogonal iff

$A = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ or $\begin{bmatrix} a & b \\ b & -a \end{bmatrix}$, where $a^2 + b^2 = 1$.