CSE

Roll No. ....

Total Pages : 3

BT-4/J-22

44151

DISCRETE MATHEMATICS

Paper–PC-CS-202A

Time : Three Hours]

[Maximum Marks : 75

Note : Attempt *five* questions in all, selecting at least *one* question from each unit.

UNIT–I

1. (a) Using mathematical induction, prove that $n^3 + 2n$ is divisible by 3.

(b) Prove that $(A \cup B)' = A' \cap B'$.

2. (a) Construct the truth table for the following statements :

(i) $\neg(p \wedge q) \wedge (\neg r)$.

(ii) $\neg(p \wedge \neg q) \vee (r)$.

(b) If the set A is finite and contains n elements, prove that the power set P(A) of the set A contains 2^n elements.

UNIT–II

3. (a) Consider relation

$$R = \{(a, b) \mid \text{length of string } a = \text{length of string } b\}$$

on the set of strings of English letters. Prove that R is an equivalence relation.

(b) Show that the inclusion relation $\subseteq$ is a partial ordering relation on the power set of a set.

4. (a) Given $A = \{1, 2, 3\}$, $B = \{a, b\}$ and $C = \{l, m, n\}$. Find each of the following sets

(i) $A \times B \times C$.

(ii) $A \times C$.

(iii) $B \times C \times A$.

(b) Define Lattice. Prove that D_{36} the set of divisors of 36 ordered by divisibility forms a lattice.

UNIT–III

5. (a) Prove that the function $f : N \to N$ defined as

$$f(n) = \begin{cases} n+1, & n \text{ is odd} \\ n-1, & n \text{ is even} \end{cases}$$

is inverse of itself.

(b) Solve : $a_n + a_{n-1} = 3n2^n$, $a_0 = 0$, using Generating function method.

6. (a) Let $f : Z \to Z$ be defined by $f[x] = 3x^3 - x$. Is this function

(i) One-to-one?

(ii) Onto?

(b) There are 280 people in the party. Without knowing anybody's birthday, what is the largest value of n for which we can prove that at least n people must have been born in the same month?

UNIT–IV

7. (a) Prove that the identity element in a group is unique.

   (b) Let G be a group and $a \in$ G. Prove that the cyclic subgroup H of G generated by a is a normal subgroup of $N(a) = \{x \in G : xa = ax\}$.

8. (a) Let P be a subgroup of a group G and let

$$Q = \{x \in G : xP = Px\}.$$

   Is Q a subgroup of G?

   (b) Let $f : (R, +) \to (R_+, \times)$ is defined as $f(x) = e^x$ for all x in R, where R $\to$ set of real numbers ond $R_+ \to$ set of positive real numbers. Prove that f is a homomorphism. Is f an isomorphism?