

Roll No.

Total Pages : 04

BT-4/M-24

44151

DISCRETE MATHEMATICS

PC-CS-202A

Time : Three Hours]

[Maximum Marks : 75

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

Unit I

1. (a) Prove that $[p \Rightarrow (q \Rightarrow r)] \Rightarrow [(p \Rightarrow q) \Rightarrow (p \Rightarrow r)]$ is a tautology. 5
- (b) Prove that : 5

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$$
- (c) Prove the following proposition by PMI : 5

$$1 + 2 + 3 + 4 + \dots + n = (n(n + 1))/2$$
2. Explain Principle of Inclusion and exclusion. A survey on a sample of 25 new cars was conducted to see which of three popular options, Air conditioning(A), radio(R), Power windows(P) were installed. The survey found that 15 had air conditioning, 4 had radio and power windows, 12 had radio, 3 had all three options, 5 had air conditioning and power windows, 2 had no options and 9 had air conditioning and radio. 15

Draw venn diagram and find the number of cars that had

- (i) Only power windows
- (ii) Only one of the options
- (iii) Air conditioning and radio but not power windows.
- (iv) Only radio.

Unit II

3. (a) Let $A = \{4, 6, 8, 10\}$ and $R = \{(4, 4), (4, 10), (6, 6), (6, 8), (8, 10)\}$. Find the transitive closure using warshall's Algorithm and also write steps of the warshall's algorithm. 8
- (b) Define Relations. Let $X = \{1, 2, 3, 4, 5, 6\}$ and R be a relation defined as $(x, y) \in R$, if and only if $x-y$ is divisible by 3. List the elements of Relation R . 7
4. (a) Consider a set $D45 = \{1, 3, 5, 9, 15, 45\}$ and let the relation \leq be the relation (divides) be a partial ordering on $D45$: 10
- (i) Determine GLB and LUB of B , B is subset of $D45$, where $B = \{9, 15, 45\}$
 - (ii) Determine GLB, LUB of B , B is subset of $D45$, where $B = \{1, 3, 5\}$
 - (iii) Draw Hasse diagram for $D45$.

- (b) Differentiate between Symmetric, Antisymmetric and Asymmetric relations with suitable examples. 5

Unit III

5. (a) Solve the recurrence relation $a_{r+2} - 5a_{r+1} + 6a_r = 2$ by using method of generating functions satisfying the initial conditions $a_0 = 1$ and $a_1 = 2$. 10

- (b) How many people at least in a group of 85 people have same initials ? 5

6. (a) Describe types of functions with suitable examples. 10

- (b) From a committee consisting of 6 men and 7 women, in how many ways can a committee be constructed, if committee consists of (i) 3 men and 4 women (ii) 4 members, which has at least one woman. 5

EXAMKIT Unit IV

7. (a) Define the following terms with suitable examples : 10

- (i) Monoid
- (ii) Abelian Group
- (iii) Ring Homomorphism
- (iv) Group.

(b) Let $G = \{-1, 0, 1\}$, verify whether G forms a group under usual addition. 5

8. (a) Consider an Algebraic system $(Q, *)$, Q is a set of rational nos. and $*$ is defined as $a*b = a + b - ab$ $\forall a, b \in Q$. Determine whether $(Q, +)$ is a group. 10

(b) Define a Semigroup. Write properties for a Semigroup. Explain with a suitable example. 5

