Roll No. .... Total Pages : 03

BT-3/D-23 43216

MATHEMATICS FOR BIG DATA AND OPTIMIZATION

BS-CS-AIDS-201A

Time : Three Hours] [Maximum Marks : 75

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

Unit I

1. (a) If $f(x)=\begin{cases}-1, & \text{for } -\pi<x<0\\ 0, & \text{for } x=0\\ 1, & \text{for } 0<x<\pi\end{cases}$ determine the Fourier series of the function $f(x)$. 7½

(b) Obtain the Fourier series for the function :

$$f(x)=\begin{cases}\pi x, & \text{for } 0\le x\le 1\\ \pi(2-x), & \text{for } 1\le x\le 2\end{cases}$$ 7½

2. (a) Find the Fourier transform of :

$$f(x)=\begin{cases}1-x^2, & \text{for } |x|\le 1\\ 0, & \text{for } |x|>1\end{cases}$$

Hence evaluate $\int_0^\infty \frac{\cos x - \sin x}{x^3} \cos\left(\frac{x}{2}\right) dx$. 7½

(b) Find the Fouriér sine and cosine transform of :

$$f(x) = \begin{cases} 1, & \text{for } 0 \le x < 2 \\ 0, & \text{for } x \ge 2 \end{cases}.$$ 7½

Unit II

3. (a) Solve the differential equation :

$$\left(xy^3 + y\right)dx + 2\left(x^2y^2 + x + y^4\right) = 0.$$ 7½

(b) Solve, by the method of variation of parameters :

$$y'' - 2y' + y = e^x \log x.$$ 7½

4. (a) Solve $y'' + 4y' + 4y = 3\sin x + 4\cos x$, $y(0) = 1$ and $y'(0) = 0$. 7½

(b) Solve :

$$3x\left(1 - x^2\right)y^2 \frac{dy}{dx} + \left(2x^2 - 1\right)y^3 = ax^3.$$ 7½

Unit III

5. (a) Using Regula-Falsi method to find the smallest positive root of the equation $x - e^{-x} = 0$. 7½

(b) Determine $f'(1.1)$ and $f''(1.1)$ from the following table : 7½

x	1.0	1.2	1.4	1.6	1.8	2.0
$f(x)$	0	0.1280	0.5440	1.2960	2.4320	4.0000

6. (a) Evaluate the integral $\int_{0.2}^{1.4}\left(\sin x - \log x + e^{x}\right)dx$ using Simpson's three-eight rule. 6

(b) Using RK method to find $y(1.2)$ in step size $h = 0.1$, given that $\frac{dy}{dx} = x^2 + y^2$ with $y(1) = 1.5$. 9

Unit IV

7. Using Kuhn-Tucker conditions, find the value(s) of β for which the point $x_1^* = 1, x_2^* = 2$ will be optimal to the problem :

Maximize : $f(x_1, x_2) = 2x_1 + \beta x_2$

Subject to :

$$g_1(x_1, x_2) = x_1^2 + x_2^2 - 5 \leq 0,$$

$$g_2(x_1, x_2) = x_1 + x_2 - 2 \leq 0.$$ 15

8. Minimize the function :

$$f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$$

with the starting point (0, 0).

15