AIDS / باب

Roll No. ..... Total Pages : 04

BT-3/D-22 **43216**

MATHEMATICS FOR BIG DATA AND OPTIMIZATION

BS-CS-AIDS-201-A

Time : Three Hours] [Maximum Marks : 75

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

Unit I

1. (a) Expand $x\sin x$ as sine series in $0 < x < \pi$. **7.5**

   (b) Find the Fourier series expansion of $f(x) = 2x - x^2$ in (0, 3) and deduce that : **7.5**

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \ldots\ldots - \infty = \frac{\pi}{12}.$$

2. (a) Using Fourier integral representation, show that : **7.5**

$$\int_0^\infty \frac{\cos x\alpha + \alpha \sin x\alpha}{1+\alpha^2} d\alpha = \begin{cases} 0, & \text{if} \quad x < 0 \\ \frac{\pi}{2}, & \text{if} \quad x = 0 \\ \pi e^{-x}, & \text{if} \quad x > 0 \end{cases}$$

(b) Show that the function $e^{-\frac{x^2}{2}}$ is self-reciprocal under Fourier transform. 7.5

Unit II

3. (a) Solve the differential equation : 7.5

$$\left(3x^2+6xy^2\right)dx+\left(6x^2y+4y^3\right)dy=0$$

(b) Solve the differential equation : 7.5

$$\frac{dy}{dx}+\frac{1}{x}y=6x^2y^3.$$

4. (a) By variation of parameter, find the solution of

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=\sin x.$$ 7.5

(b) Solve : 7.5

$$(x+1)^2\frac{d^2y}{dx^2}+(x+1)\frac{dy}{dx}+y=4\cos\log(1+x).$$

Unit III

5. (a) Using the method of False position to find the root corrected to three places of decimal of the equation

$x^3-7x-3=0.$ 7.5

(b) Find the value of sin (1.747) using the values given in the table below : 7.5

x	cos x
1.70	0.9916
1.74	0.9857
1.78	0.9781
1.82	0.9691
1.86	0.9584

6. Find an approximate value of $y(0.1)$ in the steps of 0.1, if $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$, $y(0) = 1$, $y'(0) = 0$ by Runge-Kutta Method of forth order. 15

Unit IV

7. (a) Give any *five* reasons why the study of unconstrained minimization methods is important. 7.5

(b) Define the following terms : 7.5

(i) Pattern directions

(ii) Conjugate directions

(iii) Simplex

(iv) Gradient of a function

(v) Hessian matrix of a function.

8. Minimize $f = x_1^2 + 2x_2^2 + 3x_3^2$

Subject to the constraints

$$g_1 = x_1 - x_2 - 2x_3 \leq 12,$$
$$g_2 = x_1 + 2x_2 - 3x_3 \leq 8$$

using Kuhn-Tucker conditions. **15**