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BT-3/D-22 43171

MATHEMATICS–III

BS-205A

Time : Three Hours] [Maximum Marks : 75

Note : Attempt *Five* questions in all, selecting at least *one* question from each Seciton. All questions carry equal marks.

Section A

1. (a) Discuss the convergence of the series $\sum \frac{\sqrt{n}}{\sqrt{n^2+1}} x^n$.

(b) Prove that the series $\sum \frac{x^n}{2n!}$ is convergent.

2. (a) Obtain a Fourier series expansion of the following periodic function of period :

$$f(x)=\begin{cases}\frac{1}{2}+x, & -\frac{1}{2}\le x\le 0\\ \frac{1}{2}-x, & 0<x<\frac{1}{2}\end{cases}.$$

(b) Find Fourier cosine series expansion of a periodic function $f(x)=x,\ 0<x<\pi$.

Section B

3. (a) Solve the differential equation :

$$\left(y^4+2y\right)dx+\left(xy^3+2y^4-4x\right)dy=0.$$

(b) Apply the method of variation of parameters to solve the differential equation $\frac{d^2y}{dx^2}+y=\operatorname{cosec}x$.

4. (a) Find the solution to the Bernoulli equation $\frac{dy}{dx}+\frac{y}{x}=x^2y^2$.

(b) Solve the equation :

$$\frac{d^2y}{dx^2}+2\frac{dy}{dx}+4y=\cos 2x.$$

Section C

5. (a) Find the area lying inside the cardioid $r=2(1+\cos\theta)$ and outside the circle $r=2$.

(b) Evaluate $\int_0^2\int_0^{\sqrt{2x-x^2}}\frac{xdydx}{\sqrt{x^2+y^2}}$ by changing to polar form.

6. (a) Evaluate the double integral $\int_0^{\infty}\int_0^{x}xe^{-\frac{x^2}{y}}\,dy\,dx$.

(b) Evaluate the integral $\iiint\left(x^2+y^2+z^2\right)dx\,dy\,dz$ throughout the volume of the sphere $x^2+y^2+z^2=4$.

Section D

7. (a) Find the magnetic flux generated by the magnetic field $\overrightarrow{F} = yz\hat{i} + zx\hat{j} + xy\hat{k}$ over the surface of the sphere $x^2 + y^2 + z^2 = 4$ in the first octant.

   (b) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (1, 2, 3).

8. (a) Find the directional derivative of the function $xy + yz + zx$ at the point (1, 2, 1) in the direction of the vector $\hat{i} + \hat{j} + \hat{k}$.

   (b) A fluid motion is given by $\overrightarrow{V} = r^2\overrightarrow{r}$. Show that the motion is irrotational.