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BT-4/J-22 **44182**

DISCRETE MATHEMATICS

Paper–PC-IT-204A

Time : Three Hours] [Maximum Marks : 75

Note : Attempt *five* questions in all, selecting at least *one* question from each unit. All questions carry equal marks.

UNIT–I

1. (a) Show that

$$1^2 + 2^2 + 3^2 + \ldots\ldots\ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}, n \geq 1$$

by mathematical induction.

(b) Construct the truth tables for the following statements (any *two* out of three) :

(i) $(p \to p) \to (p \to \overline{p})$.

(ii) $(p \vee \overline{q}) \to \overline{p}$.

(iii) $p \leftrightarrow (\overline{p} \vee \overline{q})$.

2. (a) If A, B, C be arbitrary sets, prove that

(i) $(A - B) - C = (A - C) - B$.

(ii) $(A - B) - C = (A - C) - (B - C)$.

(b) John made the following statements :

(i) I love Lucky.

(ii) If I love Lucky, then I also love Vivian.

Given that John told the truth or lied in both cases; determine whether John really loves Lucky.

UNIT–II

3. (a) Let A = {Fine, *Yang*] and B = {*president, vicepresident, secretary, treasurer*}. Give each of the following :

(i) $A \times B$.

(ii) $B \times A$.

(iii) $A \times A$.

(b) Let R be the relation from A to B, and let A_1 and A_2 be subsets of A. Prove that

(i) If $A_1 \subseteq A_2$ then $R(A_1) \subseteq R(A_2)$.

(ii) $R(A_1 \cup A_2) = R(A_1) \cup R(A_2)$.

4. (a) Let the relation $(x, y) \in R$, if $x \geq y$ defined on set of positive integers. Is R a partial order relation? Prove or disprove it.

(b) Suppose that R and S are relation from A to B. Prove that

(i) If $R \subseteq S$, then $R^{-1} \subseteq S^{-1}$.

(ii) $(R \cap S)^{-1} R^{-1} \cap S^{-1}$.

UNIT–III

5. (a) Prove that if $f : A \rightarrow B$ and $g : B \rightarrow C$ are one-to-one functions, then $g \, o \, f$ (composition function of g and f) is one-to-one.

(b) Find the total distinct numbers of six digits that can be formed with 0, 1, 3, 5, 7 and 9 and how many of them are divisible by 10?

6. (a) Solve the recurrence relation

$$a_n - 7a_{n-1} + 10\, a_{n-2} = 0, \; a_0 = 0 \text{ and } a_1 = 3$$

by using generating function, where $n \geq 2$.

(b) Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be functions. Show that $g \, o \, f$ (composition function of g and f) is onto, then g is onto.

UNIT–IV

7. (a) Define the following :

(i) Abelian group.

(ii) Normal subgroup.

(iii) Cyclic group.

(b) If G is a set of real numbers (non-zero) and let

$a * b = \dfrac{a.b}{2}$, show that $(G, *)$ is an Abelian group.

8. (a) Let H is a subgroup of a group G, prove that the left cosets aH, bH of H in G, are either disjoint or are identical.

(b) Let (A, + , .) be a ring such that $a \cdot a = a$ for all a in A

(i) Show that $a + a = 0$ for all a, where 0 is the additive identity.

(ii) Show that operation $\cdot$ is commutative