

Roll No.

Total Pages : 03

BT-4/M-23

44182

DISCRETE MATHEMATICS
PC-IT-204A

Time : Three Hours]

[Maximum Marks : 75

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit.

Unit I

1. (a) Give a counterexample to disprove the following propositions : 8
 - (i) $A \cup (B - C) = (A \cup B) - (A \cup C)$
 - (ii) $A \oplus (B \cap C) = (A \oplus B) \cap (A \oplus C)$.
- (b) State and prove Inclusion-Exclusion Principle. 7
2. (a) Determine whether or not each of the following is a tautology or contradiction : 6
 - (i) $\sim p \leftrightarrow (p \vee \sim p)$
 - (ii) $[(p \vee q) \wedge (\sim q)] \rightarrow p$.
- (b) What are normal forms ? Discuss its various types using suitable examples. 9

Unit II

3. (a) Prove that (D_{30}, \leq) is a lattice. Also draw a Hasse diagram of D_{30} . 7.5
- (b) If R is an equivalence relation on a set A , show that R^{-1} is also an equivalence relation on A . 7.5
4. (a) Let $\Sigma = \{a, b\}$. Define a relation R on Σ^* as : xRy if x is a prefix of y . Is R a partial order ? 7.5
- (b) How can you represent a relation in computer memory ? Explain using suitable examples. 7.5

Unit III

5. Solve the following recurrence relation using generating functions : 15
- $$S(n) - 2S(n-1) - 3S(n-2) = 0, n \geq 2 \text{ with } S(0) = 3$$
- $$\text{and } S(1) = 1.$$
6. (a) Prove that a function $f: X \rightarrow Y$ is invertible if and only if it is bijective. 7.5
- (b) Let $f(x) = ax + b$ and $g(x) = (x - b)/a$ on \mathbb{R} , where $a \neq 0$. Find $(g \circ f)(x)$ and $(f \circ g)(x)$. 7.5

Unit IV

7. (a) Prove that H , a subset of group $[G; *]$, is a subgroup. 5

(b) What do you understand by monoid, submonoid and monoid isomorphism ? Explain. For each subset, describe the submonoid that it generates : 10

(i) $\{3\}$ and $\{0\}$ in $[Z_{12}; \times_{12}]$

(ii) $\{5\}$ in $[Z_{25}; \times_{25}]$.

8. (a) Prove that group of automorphisms is a group with respect to composite of functions as the composition.

7.5

(b) Determine all zeros of $x^4 + 3x^3 + 2x + 4$ in $Z_5[x]$.

7.5