

Roll No.

Total Pages : 03

BT-4/M-24

44182

DISCRETE MATHEMATICS

Paper : PC-IT-204A

Time : Three Hours]

[Maximum Marks : 75

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

Unit I

1. (i) There are 350 farmers in a large region. 260 farm beetroot, 100 farm yams, 70 farm radish, 40 farm beetroot and radish, 40 farm yams and radish, and 30 farm beetroot and yams. Let B, Y, and R denote the set of farmers that farm beetroot, yams and radish respectively.
- (a) Determine the number of farmers that farm beetroot, yams, and radish.
- (b) Determine number of farmers that farm radish and yams but not beetroot
- (c) Draw Venn diagram.

10

(ii) Prove that : 5

$$A \cup B = (A \cap B^c) \cup (A^c \cup B) \cup (A \cap B)$$

2. (i) Differentiate Universal and Existential Quantifiers with suitable examples. Show that $P \rightarrow (Q \rightarrow R)$ is equivalent to $(P \wedge Q) \rightarrow R$. 7.5

(ii) Differentiate DNF and CNF with suitable examples. Obtain DNF of $(P \rightarrow Q) \wedge (\neg P \wedge Q)$. 7.5

Unit II

3. Draw the Hasse diagram of the poset $(D_{20}, |)$ and find its least and greatest elements, where D_{20} is the set of positive divisors of 20. Determine GLB and LUB of X , where $X = \{1, 2, 4\}$. Also check whether D_{20} is lattice or not ? 15

4. Define Relation. Explain various types of relation. Give an example of a relations R_1, R_2 and R_3 on set $A = \{a, b, c, d\}$ having property : 15

- (i) R_1 is irreflexive and antisymmetric
- (ii) R_2 is asymmetric and antisymmetric.
- (iii) R_3 is asymmetric but not transitive.

Unit III

5. Define functions. Explain various types of functions with suitable example. Show that function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = 3x + 4$ for all $x \in \mathbb{R}$ is one-one onto. 15
6. (i) Solve the recurrence relation $a_{r+2} - 3a_{r+1} + 2a_r = 0$ by method of generating functions with initial conditions $a_0 = 2$ and $a_1 = 3$. 10
- (ii) How many people atleast in a group of 85 people have same initials ? 5

Unit IV

7. Define following with suitable examples : 15
- (i) Ring
 - (ii) Monoid
 - (iii) Semigroup
 - (iv) Coset
 - (v) Normal subgroup.
8. Define an Abelian Group. Explain properties of Abelian Group. consider an Algebraic system $(G, *)$, where G is set of real numbers and $*$ is a binary operation defined by $a * b = ab/4$, show that $(G, *)$ is an Abelian Group. 15